18. A. E. Voitenko and V. I. Kirko, "The efficiency of explosion plasma compressor," Fiz. Goreniya Vzryva, No. 6, 956-958 (1975).

UNIDIRECTIONAL NONSTATIONARY FLOW OF INSTANTANEOUSLY HEATED GAS FROM A CYLINDER FOR DIFFERENT POSITIONS OF THE HEATED ZONE

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The finite-difference method and one-dimensional approximation are used to consider the processes of decay of an initial pressure discontinuity for the individual cases of its position in a bounded region.

The classical problem of one-sided and two-sided nonstationary flows of gas from a cylinder as a result of an increased initial pressure in the cylinder is encountered in various modifications in the study of gasdynamic processes in many technical devices, e.g., in shock tubes.

One encounters physical situations in which the residual pressure in the cylinder is created by instantaneous heating of the gas. The development of the nonstationary process depends here, in particular, on the relationship between the lengths of the cylinder and of the heated zone, and also on the position of this zone.

In the present work we numerically study the decay of a pressure discontinuity in a cylinder of finite length. One end of the cylinder is open to the atmosphere at finite pressure and the other end is insulated at various positions of the heated zone, which can take up the whole volume, or part of the volume near the open end, near the closed end, or in the center of the cylinder.

The problem was solved in the one-dimensional approximation, and the processes were assumed to be adiabatic. However, instead of the adiabatic curve, we used the general equation of state. This is because, in the thermodynamic viewpoint, the system in question is not closed, i.e., the parameters of the gas flowing into the system from the external medium at the suction stage do not satisfy the equation of the adiabatic curve for the initial working substance. In the equation of motion we took into account the force of viscous friction averaged over the cross section. The local resistance at the open end was taken into account in the form of a boundary condition — as a pressure drop proportional to the square of the outflow velocity. The initial distributions of pressure and temperature were determined by the position of the heated zone. The system of equations that describes the nonstationary process has the form

$$
\begin{gathered}
\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=-\frac{1}{\rho}\frac{\partial p}{\partial x}-\frac{\tau_0\chi}{\rho F},\\
\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)=0,\\
\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+(k-1)T\frac{\partial u}{\partial x}=0,\\
p/\rho T=\mathrm{const},\ \mathrm{const}=p_1/\rho_0 T_1.
\end{gathered}
\tag{1}
$$

Here T_1, p_1, ρ_0 are the initial values of temperature, pressure, and density of the gas, respectively.

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The form of τ_0 will be chosen according to [1] (p. 97):

$$\tau_0 = \frac{\lambda}{8}\rho u^2,$$

and the friction coefficient λ is assumed, as in the case of stationary motion, to be dependent on the Reynolds number Re according to the relation

$$\lambda = 0{,}00332 + 0{,}221 \cdot \mathrm{Re}^{-0{,}237},$$

$$\mathrm{Re} = \frac{4r_h u\rho}{\mu}, \quad r_r = F/\chi.$$

In the system (1), we go over to dimensionless quantities by taking the quantities L, a_0, p_0, ρ_0, L/a_0, T_0 as the scales of length, velocity, pressure, density, time, and temperature, respectively. By writing the system (1) for the dimensionless quantities (for which we retain the previous notation) we obtain

$$\begin{gathered}
\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{1}{k\rho}\frac{\partial p}{\partial x} = -f(u), \\
\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + kp\frac{\partial u}{\partial x} = 0, \\
\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + (k-1)T\frac{\partial u}{\partial x} = 0, \\
p = \rho T, \quad f(u) = \frac{\lambda L}{8r_h}u^2.
\end{gathered} \tag{2}$$

In writing the equation of state, we took into account the equality of the dimensional ratios p_1/p_0, T_1/T_0 during an isochoric heating. By taking r_h = 0.15 m, L = 4 m, μ = 1.61$\cdot$ 10^{-5} N$\cdot$sec/m^2, the right-hand side of the equation of motion will be written in the following form:

$$f(u) = 0{,}0111\,[1 + 1{,}25(\rho u)^{-0{,}237}]\,u^2. \tag{3}$$

We shall now write the boundary conditions for the system (2):

1) At the closed end,

$$u = 0, \ T = T_1 \ \text{for} \ x = 0, \ t > 0, \tag{4}$$

if the heated zone is close to the closed end, and

$$u = 0, \ T = T_0 \ \text{for} \ x = 0, \ t > 0 \tag{4'}$$

in all other cases.

2) At the open end

$$p = p_0, \ T = T_0 \ \text{for} \ x = 1, \ t > 0. \tag{5}$$

The initial conditions for the functions of the system (2) will be the values

$$\begin{gathered}
u = 0, \ p = p_1, \ T = T_1 \ \text{for} \ t = 0, \ x_0 \leqslant x \leqslant x_1, \\
u = 0, \ p = p_0, \ T = T_0 \ \text{for} \ t = 0, \ \begin{cases} x_1 < x \leqslant 1, \\ 0 \leqslant x < x_0. \end{cases}
\end{gathered} \tag{6}$$

In agreement with the general theory of solutions of the systems of hyperbolic equations [2, 3], for a numerical solution the system must be transformed to an invariant form. We shall omit the derivation and write down the equivalent system with Riemann invariants:

$$\begin{gathered}
\frac{\partial X}{\partial t} + \left(\frac{X+Y}{2} + \sqrt{T}\right)\frac{\partial X}{\partial x} = -f\left(\frac{X+Y}{2}\right), \\
\frac{\partial Y}{\partial t} + \left(\frac{X+Y}{2} - \sqrt{T}\right)\frac{\partial Y}{\partial x} = -f\left(\frac{X+Y}{2}\right),
\end{gathered} \tag{7}$$

$$\frac{\partial Z}{\partial t}+\frac{X+Y}{2}\frac{\partial Z}{\partial x}=0.$$

Here the Riemann invariants are

$$X=u+\int_{p_1}^{p}\frac{dp}{k\sqrt{\rho p}};\; Y=u-\int_{p_1}^{p}\frac{dp}{k\sqrt{\rho p}};\; Z=\ln(T/p^{\frac{k-1}{k}}). \tag{8}$$

Using (4) and (5), the boundary conditions for the functions (8) will be written in the form:

1) At the closed end,

$$X+Y=0;\; Z=\ln(T_1/p_1^{\frac{k-1}{k}}) \text{ for } x=0,\; t>0, \tag{9}$$

if the heated zone is very near the closed end, and

$$X+Y=0,\; Z=0 \text{ for } x=0;\; t>0$$

in all other cases.

2) At the open end,

$$X-Y=\int_{p_1}^{p_0}\frac{dp}{k\sqrt{\rho p}};\; Z=0 \text{ for } x=1,\; t>0. \tag{10}$$

The initial conditions have the form

$$X=Y=\int_{p_1}^{p_0}\frac{dp}{k\sqrt{\rho p}};\; Z=0 \text{ for } t=0,\; \begin{cases}0\leqslant x<1,\\ x_1<x<1,\end{cases}$$

$$X=Y=0,\; Z=\ln(T_1/p_1^{\frac{k-1}{k}}) \text{ for } t=0,\; x_0\leqslant x\leqslant x_1. \tag{11}$$

Since (7)-(11) is a problem with mixed boundary conditions, it is necessary to match the boundary and initial conditions to ensure the existence of a continuous solution in the internal points of the region $t>0$, $0\leqslant x\leqslant 1$.

For subsonic flows $\sqrt{T}>(X+Y)/2$, and the direction of the characteristics for the invariant X will always be positive, for Y it will always be negative, and for the invariant Z the direction of the characteristics at some moment of time will, for each coordinate, change sign in accordance with the sign of the characteristic velocity $u=(X+Y)/2$. The matching conditions at the open end are satisfied automatically, since the initial values of the invariants at point $x=0$ satisfy the boundary conditions. This is also true for the open end if the heated zone is not completely adjacent to it. Since at the first stage of outflow the characteristic velocity for Z is positive ($u>0$), one must specify here the boundary condition from the left ($x=0$). Consequently one does not need to match the conditions for this invariant at the open end ($x=1$). It is therefore necessary to match only the initial values of the invariants X and Y at the point $x=1$, with the first condition (10) for the combination $X-Y$. Since for X one does not need to specify the boundary condition from the right, it is natural to assume that it is continuous by including the boundary point $x=1$, i.e., $X=0$ for $x=1$, $t=0$. The requirement of matching then leads to the discontinuity of the initial value of Y at point $x=0$. The discontinuity is determined from the first condition (10) in the form

$$X=0;\; Y=-2\int_{p_1}^{p_0}\frac{dp}{k\sqrt{\rho p}} \quad \text{for } t=0,\; x=1. \tag{12}$$

The discontinuity of the initial value of the invariant Y at the boundary point at the open end is a consequence of the pressure discontinuity at the initial moment, which leads to a sudden occurrence of outflow velocity u_0. The magnitude of the velocity u_0, which remains constant at the boundary of the rarefaction wave until the moment of its reflection from the end wall, is determined by the formula

$$u_0=-\int_{p_1}^{p_0}\frac{dp}{k\sqrt{\rho p}}. \tag{13}$$

To take into account the local resistance at the open end, we shall assume that the characteristics of resistances established for stationary motion of the liquid in tubes with a sudden pressure variation are also retained for the nonstationary motion ([1], p. 178). The pressure loss by the local resistance is determined by the relation

$$\Delta p = p_0' - p_0 = \xi\rho \left.\frac{u^2}{2}\right|_{x=1}.$$

The coefficient of local resistance ξ can be taken equal to unity when the outflow is into the atmosphere. We then obtain

$$p_0' = p_0 + \rho \left.\frac{u^2}{2}\right|_{x=1}, \tag{14}$$

and in Eqs. (10, 12) the upper limit in the integral should be taken as p_0' instead of p_0.

The problem (7)-(11) was solved numerically by using an explicit scheme of the finite-difference approximation of Eq. (7). We do not give the recurrence finite-difference relations for the stepwise determination of the quantities in the nodes. These problems were discussed in detail in [5] for the adiabatic outflow (the situation when the third equation in the system (1) is absent). We shall only note the important features of this problem.

The requirement that the system of finite-difference equations satisfies the conditions of Courant, Fridriechs, and Levy ([4], Sec. 24) leads to the fact that the third equation in the system (7) must be written in the form of two finite-difference schemes which correspond to the positive and negative values of characteristic velocity $u = (X + Y)/2$ for the invariant Z. In the first case, the derivative $\partial Z/\partial x$ at the instantaneous point i should be replaced by the backward difference ratio $(Z_i - Z_{i-1})/h$ and one should use the left-hand boundary condition for Z. In the second case, the derivative should be replaced by the forward difference ratio $(Z_{i+1} - Z_i)/h$ and one should use the right-hand boundary condition. For the convergence of the finite difference scheme it is also necessary that the ratio of the steps τ and h satisfies the condition $\alpha\tau/h < 1$, where τ is the time step, h is the coordinate step, and α is the largest value of the three characteristic velocities which can be reached during the outflow process.

It is easily seen that for α one can take the value $\alpha = u_0 + \sqrt{T}$; we have here $u_0 > 0$.

The recurrence difference equations which approximate the first two Eqs. (7) contain the quantity Z as a parameter in terms of the temperature T. Consequently, a change of the quantities X and Y when one goes from one time layer to another takes place at constant Z. Therefore, in Eqs. (8) one can calculate the integral by expressing ρ in terms of p/T, and T in the third Eq. (8) in terms of p and Z, and assuming that Z is a parameter. As a result, we obtain the following relations:

$$X = u + \frac{2\exp(Z/2)}{k-1}\left(p^{\frac{k-1}{2k}} - p_1^{\frac{k-1}{2k}}\right);$$
$$Y = u - \frac{2\exp(Z/2)}{k-1}\left(p^{\frac{k-1}{2k}} - p_1^{\frac{k-1}{2k}}\right); \tag{15}$$
$$Z = \ln\left(T/p^{\frac{k-1}{k}}\right).$$

Hence, after obtaining the numerical results one can easily obtain the formulas for the transition from the invariants to the initial variables u, p, T, ρ.

Taking into account Eqs. (15), we obtain the following expression for the velocity u_0 during the initial outflow period:

$$u_0 = \frac{2\sqrt{T_1}}{k-1}\left[1 - (p_0/p_1)^{\frac{k-1}{2k}}\right],$$

according to which, for the numerical values $k = 1.4$, $p_0 = 1$, $p_1 = T_1 = 1.6$, the quantity u_0 is equal to 0.41.

Given numerical values of the quantities $\alpha \approx 1.7$, the condition of convergence has the form $\tau/h < 0.6$. In the calculations we used $h = 0.01$, $\tau = 0.001$, which satisfies the above condition.

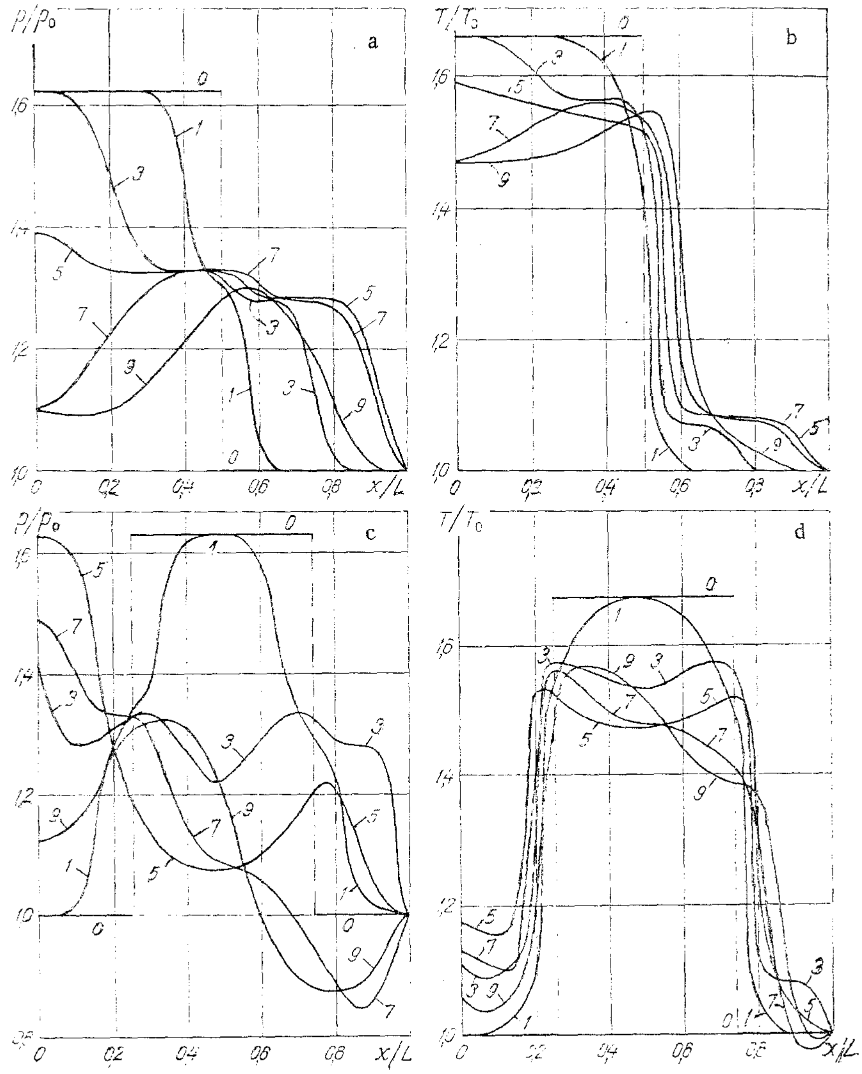

Fig. 1. The distributions of pressure (a, c) and temperature (b, d) along the cylinder for consecutive moments of time.

In the analysis of the results it is necessary to bear in mind that using an explicit finite-difference scheme leads to the numerical "smoothing out" of the initial data, which is associated with the approximate viscosity $(h - \tau)/2$ in this scheme ([4], Sec. 25). For this reason, the discontinuities of pressure and temperature during the propagation of waves are not sharply pronounced but are smeared over space, which resembles the pattern for the rarefaction waves, where the smearing of the initial discontinuities of pressure and temperature is determined by the physical essence of the process.

For the case when the heated zone takes up the whole volume of the cylinder, the calculations were carried out in three variants. In the first variant, we did not take into account friction at the walls and the forces of local resistance at the open end. During the outflow stage (until the moment when the minimum pressure ($\sim$0.6) is established in the volume, and the external medium is beginning to be sucked in), the distribution of the gasdynamic quantities over time and space is exactly the same as when one uses the equation of the adiabatic curve [5]. However, during the suction stage they are considerably different. For example, the maximum value of pressure in the cylinder and of the temperature of the residue of the original gas after the completion of suction, considerably exceed their initial values (for the initial pressure and temperature of 1.6, these quantities will be $\sim$1.89 and $\sim$1.67, respectively). At the first sight, this result is paradoxical. However, it does not contradict the law of energy conservation. The total energy of a unit mass of the substance in the cylinder after the completion of the compression stage does not exceed its initial value. An increase of the potential energy of the pressure takes place as a result of decreasing the internal energy of the system, since the temperature of the gas which enters

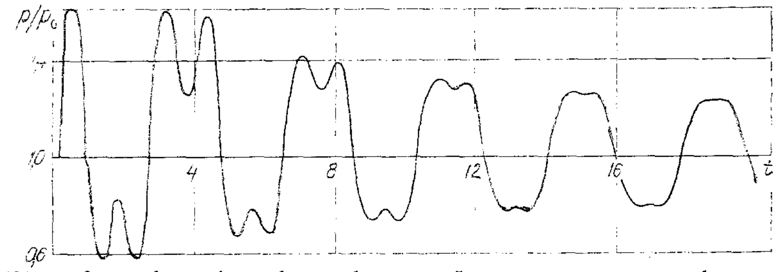

Fig. 2. The time dependence of pressure near the closed end. The region of heat dissipation takes up 2/3 of the volume of the cylinder near the open end.

from outside and displaces the outflowing part of the original gas is below the temperature of the gas which remains in the cylinder after expansion.

In the second variant, we took into account the resistance of friction of the gas at the walls, and the local resistance at the open end of the cylinder was neglected. As was to be expected, the results differ weakly from the corresponding results neglecting friction (maximum values of gasdynamic quantities during a period differ only in the third significant figure).

In the third variant, we included both the resistance of friction of the gas at the walls as well as the forces of local resistance at the open end. It was established that in this case one has a considerable damping of the process with time. To clarify the character of this damping, we calculated the distributions of gasdynamic quantities for a sufficiently large time interval equal to 25 characteristic units (1 characteristic unit = $L/a_0 \approx 0.012$ sec). From these distributions we obtained the dependence of pressure on time near the closed end (the structure of this curve is similar to the graph in Fig. 2, where the valleys are replaced by peaks). For the initial pressure discontinuity of 0.6, after the first period (~3.75 characteristic units) the maximum pressure was 1.63, and when the friction was neglected, this value was equal to 1.89. The losses were equal to $\Delta p = 0.26$.

During eight periods, the pressure is reduced to approximately 60% of the initial discontinuity. These results agree qualitatively with the experimental data in a shock tube [6]. The temperature distribution makes it possible to obtain information about the position of the contact surface between the heated and external gases. We found that the contact surface in the present problem reaches the point ~0.6, i.e., the external medium reaches, through the open end, the depth ~0.4 of the length of the cylinder.

Figures 1a,b show the distributions of pressure and temperature at the early stage of the outflow process for the case when the heated zone takes up half of the volume of the cylinder near the closed end. The number 0 denotes the initial values of the quantities. The graphs correspond to the moments of time $t_n = n\Delta t$, where n is the number of the curve (n = 1, 3, 5, 7, 9), and $\Delta t = 0.08$ characteristic units. Figure 1a clearly shows the propagation of the rarefaction wave to the left of the center of the cylinder towards its closed end (the upper parts of graphs 1, 3, and 5) and of the compression wave to the right of the center of the cylinder towards its open end (the lower parts of graphs 1, 3, and 5), and also the backward propagation of the reflected rarefaction waves (graphs 7 and 9). It is seen from Fig. 1b that the contact surface in the present time interval moves to the right from the center of the cylinder towards its open end.

Using the same notation as in Figs. 1a and b, Figs. 1c and d show the distributions of pressure and temperature at an early stage of the outflow process when the heated zone is positioned at the center of the cylinder. It is seen from Fig. 1c that the evolution pattern of the pressure distribution is considerably more complex than in the previous case. This is a result of the appearance (in the region of the cylinder which is free from heating) of compression waves, the interaction of the rarefaction and compression waves, and of the direct waves and waves reflected from the closed end.

In the case when the heated zone takes up 2/3 of the volume of the cylinder near the open end, the calculations of distributions of pressure, velocity, and temperature were carried out until the moment of time equal to 25 characteristic units. It was shown that the contact surface which separates the heated gas from the cold gas near the closed end reaches only up to the point ~0.22. Therefore, in a large part of the region which is free from heating, the changes of temperature during the outflow process are negligible. The contact surface near the open end shows that during the suction process the external medium reaches the depth ~0.38 of the length of the cylinder. The distribution of pressure was used to determine the dependence of pressure on time, which is shown in Fig. 2.

NOTATION

x is the coordinate along the axis of the cylinder; t, time; u, velocity; p, pressure; ρ, density; k, exponent of the adiabatic curve; χ, wetted perimeter of the cylinder; F, cross-sectional area of the cylinder; τ_0, force of friction of the liquid against the cylinder walls per unit area; λ, friction coefficient; r_h, hydraulic radius of the cylinder; μ, coefficient of dynamical viscosity; Re, Reynolds number; and a, speed of sound. The subscript 0 refers to the conditions of the external medium, and 1 to the perturbed conditions.

LITERATURE CITED

1. I. P. Ginzburg, Applied Gas and Hydrodynamics [in Russian], Leningrad State University Press, Leningrad (1958).
2. S. K. Godunov, Equations of Mathematical Physics [in Russian], Nauka, Moscow (1971).
3. B. L. Rozhdestvenskii and N. N. Yanenko, Systems of Quasilinear Equations and Their Application to Gas Dynamics [in Russian], Nauka, Moscow (1968).
4. S. K. Godunov and V. S. Ryaben'kii, Difference Schemes [in Russian], Nauka, Moscow (1973).
5. É. T. Bruk-Levinson and E. A. Romashko, in: Physics and Technology of Aerothermooptical Methods of Control and Diagnosis of Laser Radiation [in Russian], ITMO Akad. Nauk. Belo. SSR, Minsk (1981), pp. 76-87.
6. M. J. Levy and J. H. Potter, "Gas flow in a rarefaction tube," Nav. Eng. J., 941-950, December (1964).

HEAT TRANSFER BETWEEN THREE MEDIA IN TRIPLE COUNTERCURRENT PIPE FLOW

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UDC 536.27

A method is discussed for calculating the thermodynamic characteristics associated with the interaction of three fluid flows in pipes. Analytical relations are derived for the case of triple concentric countercurrent flow.

The calculation of heat transfer between two media flowing in the same or opposite directions does not present any difficulties. Recently, however, there has been a growing number of problems in which it is required to calculate heat transfer in the simultaneous interaction of three flows, but the literature does not offer analytical solutions for determining the temperature profile along the flow axis, the quantity of heat transmitted across the separating surface, and other characteristics. The number of combinations along the relative direction of motion of the media and in the direction of heat transfer can be enormous in this case. The most complicated situation in this class of problems is triple concentric countercurrent flow of the media.

In particular, e.g., the recent efforts aimed at intensifying petroleum recovery have created the important problem of supplying heat to oil-bearing strata at great depths. One of the more promising methods of solving this problem is to create a deep underground steam generator of adequate output, situated in the downhole zone, with injection of the generated steam into the oil stratum. The specific attributes of this problem are that, first, it is required to lower the steam generator through a drivepipe with a diameter of 150-200 mm and, second, to supply the steam generator with air, fuel, and water and to exhaust the combustion products to the surface of the earth. At the same time, it is necessary to maintain the

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